Black holes and gravitational waves in string cosmology

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Pre–big bang models of inflation based on string cosmology produce a stochastic gravitational wave background whose spectrum grows with decreasing wavelength, and which may be detectable using interferometers such as LIGO. We point out that the gravitational wave spectrum is closely tied to the density perturbation spectrum, and that the condition for producing observable gravitational waves is very similar to that for producing an observable density of primordial black holes. Detection of both would provide strong support to the string cosmology scenario.

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I. INTRODUCTION

The pre–big bang string cosmology scenario is a novel way of producing inflation which capitalizes on the kinetic energy of a scalar field, the dilaton, rather than the potential energy as in conventional models [1]. It possesses two phases, the first known as the dilaton phase and the second the string phase. (For a recent review, see e.g. Ref. [2]). During the dilaton phase, the space–time curvature and gravitational coupling both grow with time until the former reaches the string scale, although the latter may still be small. At this stage non–perturbative effects become important and the universe enters the string phase, where the dynamics are much less certain. Eventually the string phase gives way to the standard hot big bang picture.

Scalar (density) and tensor (gravitational wave) perturbations are generated in the universe during the dilaton phase, and can be calculated using standard techniques [3,4]. A much-advertized prediction of the dilaton phase is that the spectrum of gravitational waves is steeply rising to short scales, in contrast to potential-driven inflation models where it must decrease as one looks to shorter scales. In the latter case, the Cosmic Background Explorer (COBE) observations already guarantee that the stochastic gravitational wave background lies orders of magnitude below the sensitivity of even advanced versions of the Laser Interferometric Gravitational Wave Observatory (LIGO) currently under construction [5]. In the dilaton phase, however, the spectrum rises as k^3 (k being the comoving wavenumber), which has led several authors to suggest that LIGO may be able to detect these perturbations [4].

However, the adiabatic density perturbations that are also produced during the dilaton phase have an extremely similar amplitude on a given scale to that of the gravitational waves [6,7]. We define the scalar and tensor amplitudes $A_{\rm S}^2$ and $A_{\rm T}^2$ as in Ref. [8] (note that $A_{\rm S}$ is the same as $\delta_{\rm H}$ of Ref. [9] and represents the density contrast at Hubble–radius–crossing during a matter–dominated era). The present energy density of gravitational waves is given from the initial amplitude by [10]

$$\Omega_{\rm gw}(k) = \frac{25}{6}\frac{A_{\rm T}^2}{z_{\rm eq}}\,, \tag{1}$$

where $z_{\rm eq} = 24\,000\,\Omega_0 h^2$ is the redshift of matter radiation equality. Here Ω_0 and h are the present density parameter and Hubble parameter, in the usual units.

We write the tensor to scalar ratio as

$$\frac{A_{\rm T}^2}{A_{\rm S}^2} = \epsilon\,. \tag{2}$$

In a conventional inflation model ϵ is the usual slow-roll parameter [11], and in the slow–roll approximation it is bounded, $0 < \epsilon < 1$, and normally much less than one.

In the dilaton phase of string cosmology, however, ϵ equals 3. Although the dilaton phase is far from the usual slow-roll limit, the general relativistic result that $\epsilon = 1/p$ for power-law inflation/deflation, where the scale factor $a \propto t^p$, still holds. This result is exact because the scalar and tensor perturbations obey the same evolution equation and the ratio is then fixed by their normalization as adiabatic vacuum fluctuations on small scales. In conventional power-law inflation $p > 1$ and thus $\epsilon < 1$, but in the Einstein frame of low-energy string theory the

dilaton phase corresponds to a collapsing universe with $p = 1/3$ [12]. This is a generic prediction for adiabatic density perturbations in any model which is conformally equivalent to a collapsing universe in Einstein gravity, as this represents massless fields with a maximally stiff equation of state dominating the energy density as the scale factor $a \to 0$. We will discuss the possible effect of non-adiabatic perturbations later.

Combining Eqs. (1) and (2), we find that on *any* scale k

$$A_{\rm S}^2 = \frac{1}{3}\, A_{\rm T}^2 = 2 \times 10^{-3}\, \frac{\Omega_{\rm gw}}{10^{-6}}\, \Omega_0 h^2 \,. \tag{3}$$

Thus, both $A_{\rm S}^2$ and $A_{\rm T}^2$ exhibit an increase as k^3 with wavenumber in the pre-big bang scenario [6].

II. GRAVITATIONAL WAVES

A very detailed analysis of the detectability of the gravitational waves by LIGO has been made by Allen and Brustein [13]. LIGO is sensitive to frequencies around $f \approx 100$Hz. During the dilaton phase $\Omega_{\rm gw}$ grows as k^3, and this portion of the spectrum is characterized by the frequency, $f_{\rm s}$, and fractional energy density, $\Omega_{\rm gw}^{\rm s}$, at the point where the dilaton phase ends. Note that the frequency f and wavenumber k are interchangeable, since we set $c = 1$. If the string phase is inflationary, then higher frequency gravitational waves will be produced, but our understanding of the generation of perturbations is much less certain. Allen and Brustein take the spectrum to have an arbitrary slope β in this region [13]. During an inflationary string phase, all scalar and tensor perturbations that exited during the dilaton phase will remain beyond the Hubble radius. Thus, in what follows, we assume that Eq. (3) remains valid over those scales where $f < f_{\rm s}$ and, furthermore, that the frequencies accessible to LIGO lie in this regime, i.e., that these modes exited the Hubble radius during the dilaton phase.

Allen and Brustein demonstrate that much of the parameter space where a stochastic gravitational wave background could be detectable by the initial LIGO configuration is already excluded by primordial nucleosynthesis bounds [13,14]. From here on, therefore, we focus on the advanced LIGO configuration. For a frequency at the end of the dilaton phase around 100 Hz, advanced LIGO can probe to $\Omega_{\rm gw}^{\rm s} \sim 10^{-9}$. For comparison, the nucleosynthesis bound is $\Omega_{\rm gw} \lesssim 5 \times 10^{-5}$.

III. DENSITY PERTURBATIONS AND BLACK HOLES

Density perturbations whose amplitude is of order unity when they re-enter the Hubble radius can immediately collapse to form primordial black holes. Because black holes redshift more slowly than the radiation, which is assumed dominant, even a very modest initial fraction (perhaps 10^{-20} by mass) can be observationally constrained. Hence, any black holes which form correspond to high-sigma fluctuations in the density field, whose mean square perturbation must therefore be well below unity.

Assuming the standard cosmology (we shall examine alternatives later), the epoch during the radiation–dominated era when a comoving scale f_* equals the Hubble scale is determined by

$$\frac{f_*}{f_0} = \frac{H_* a_*}{H_0 a_0} \approx \frac{T_*}{T_{\rm eq}}\, z_{\rm eq}^{1/2} \tag{4}$$

where $f_0 = a_0 H_0 = 3h \times 10^{-18}$ Hz is the mode that is just re-entering the Hubble radius today. Since $T_{\rm eq} = 24\,000\, \Omega_0 h^2\, T_0 \approx 1$eV, we have

$$\frac{f_*}{100\,{\rm Hz}} \approx \frac{T_*}{10^9\,{\rm GeV}} \,. \tag{5}$$

The mass of black holes forming from perturbations that collapse immediately after re-entry is given by the horizon mass at that time, up to a numerical factor of order unity. In a radiation-dominated universe, this is given approximately by $M_{\rm hor} \approx 10^{32} (T/{\rm GeV})^{-2}$g, and the black hole mass for a given mode f_* is therefore

$$M \approx 10^{14} \left(\frac{100\,{\rm Hz}}{f_*} \right)^2 {\rm g} \,. \tag{6}$$

Whether or not black holes of mass M form is governed by the dispersion σ of the matter distribution smoothed on the length scale R giving that horizon mass. The dispersion is defined as (see e.g. Ref. [9])

$$\sigma^2(R,t) = \left(\frac{10}{9}\right)^2 \int_0^\infty \left(\frac{k}{aH}\right)^4 A_{\rm S}^2(k)\, W^2(kR)\, \frac{dk}{k} \,, \tag{7}$$

where $A_{\rm S}$ is related to $\Omega_{\rm gw}$ by Eq. (3), the time-dependence is carried by the aH factor, and the prefactor appears because we are considering radiation domination rather than the usual matter domination. We take the smoothing window $W(kR)$ to be a gaussian; for an $A_{\rm S}^2 \propto k^3$ spectrum the top-hat filtered dispersion remains dominated by the shortest scales rather than the smoothing scale and so such smoothing is unsuitable.

We first assume that there are no scalar perturbations generated during the string phase (the 'dilaton only' scenario in the language of Allen and Brustein [13]), so the spectrum vanishes for $f > f_{\rm s}$. The steeply-rising spectrum will guarantee that only modes close to $f_{\rm s}$ can give significant black hole production. We measure R in units of $k_{\rm s}^{-1}$, and consider the dispersion $\sigma_{\rm hor}$ when that scale R crosses the horizon, so that $aH = 1/R$. Substituting in from Eq. (3) gives

$$\sigma_{\rm hor}^2(k_{\rm s}R) = 2 \times 10^{-3}\, \Omega_0 h^2\, \frac{\Omega_{\rm gw}^{\rm s}}{10^{-6}} \times (k_{\rm s}R)^4 \int_0^1 \tilde{k}^6 W^2(\tilde{k} k_{\rm s} R)\, d\tilde{k} \,. \tag{8}$$

For $k_sR \ll 1$ (small scales) this is small due to the prefactor, as the perturbations contributing to σ are on longer scales than the horizon and have not had time to grow to their horizon-crossing value. For $k_sR \gg 1$ (large scales) this is small as the dominant short-scale perturbations have been smoothed out. Therefore $\sigma_{\rm hor}$ peaks for $R \simeq k_s^{-1}$, and it is on this scale that black holes predominantly form.

There are some uncertainties in the exact parameters required for black hole formation, though these are not particularly important for our calculations. The usual criterion during radiation domination is that black holes form in any region with density contrast greater than a threshold $\delta_c = 1/3$ when they enter the horizon, and that the corresponding black hole mass is 0.2 times the horizon mass (see e.g. Ref. [15]).

Given the dispersion σ, the fraction of the Universe in regions with density contrast exceeding δ_c is given by the integral over the tail of the gaussian, yielding a mass fraction

$$\beta = \mathrm{erfc}\left(\frac{\delta_c}{\sqrt{2}\,\sigma_{\rm hor}(k_sR)}\right)\,, \tag{9}$$

where 'erfc' is the complementary error function

$$\mathrm{erfc}(x) \equiv \frac{2}{\sqrt{\pi}}\int_x^\infty \exp(-u^2)\,du\,. \tag{10}$$

This expression is familiar from Press–Schechter theory in large-scale structure studies, and gives the fraction of the total mass in black holes with a mass greater than or equal to the smoothing mass.*

Black holes of mass 10^9g evaporate around nucleosynthesis, while those of mass 5×10^{14}g are evaporating at the present epoch. Black holes within this mass range are constrained by a range of different observations, summarized in Refs. [16,17]. Those of mass above 5×10^{14}g have negligible evaporation, and are constrained by their contribution to the present total matter density. In all cases, the initial mass fraction of black holes must be tiny, since it grows in proportion to the scale factor during the long radiation-dominated epoch.

Equation (6) shows that primordial black holes which are evaporating at the present epoch are formed from density perturbations with the same comoving wavelength as the gravitational waves which LIGO hopes to detect. As the error function depends so strongly on its argument, we can adopt an extremely qualitative view of the observations; namely, that for the standard cosmology the initial mass fraction β should be no more than 10^{-20} on those scales [16,17]. As $\mathrm{erfc}^{-1}(10^{-20}) \simeq 6$ the observational constraint† corresponds to $\sigma_{\rm hor} < 0.04$.

To convert this into a constraint on $\Omega_{\rm gw}^{\rm s}$, note that for a gaussian smoothing, $W(y) = \exp(-y^2/2)$,

$$\max\left\{x^4\int_0^1 \tilde{k}^6 W^2(\tilde{k}x)\,d\tilde{k}\right\} \simeq 0.15 \quad \text{at } x \simeq 1.74\,. \tag{11}$$

Thus, the main black hole formation corresponds to $R = 1.74/k_s$, and the fraction of the total mass in black holes above the corresponding formation mass (in practice dominated by black holes close to this mass) is given by Eq. (9). Hence the gravitational wave amplitude leading to a black hole density at the current observational limit is, from Eq. (8),

$$\Omega_{\rm gw}^{\rm s} = \frac{5\times 10^{-6}}{\Omega_0 h^2}\,. \tag{12}$$

For plausible values of $\Omega_0 h^2$, this is a little below the bound on gravitational waves from nucleosynthesis, which in this dilaton-only scenario is $\Omega_{\rm gw}^{\rm s} < 5\times 10^{-5}$ [13,14].

IV. COMPLICATIONS

A. Non-adiabatic perturbations

In taking the ratio ϵ between the tensor and scalar perturbations to be exactly 3, we have assumed that the scalar curvature perturbations are due to purely adiabatic fluctuations. This is a natural assumption in many conventional models of inflation where fluctuations in only one scalar field determine the final amplitude of density perturbations. However, in the low energy effective action there are many massless fields, with associated spectra of fluctuations [18]. This may be very important for calculating the perturbations on large scales where the fluctuations in the dilaton, and other fields minimally coupled in the Einstein frame, are strongly suppressed. Indeed, if the pre–big bang era is to be able to generate seed perturbations for large-scale structure, then non-adiabatic perturbations in other scalar fields, such as the axion fields, must play a significant role [18].

A gaussian spectrum of non-adiabatic perturbations, uncorrelated with the original adiabatic spectrum, can only add to the overall scalar curvature perturbation power spectrum [19,20]. We can represent their effect by introducing an effective value of $\epsilon_{\rm eff} < 3$ during the pre–big bang phase. The maximum density of gravitational

*We have included the factor two multiplier on the right-hand side on Eq. (9) which is added in the large-scale structure context to ensure underdense regions contribute to gravitationally-bound objects. Inclusion or otherwise of this factor has a completely negligible impact on our results.

†For comparison $\mathrm{erfc}^{-1}(10^{-10}) \simeq 4.5$, from which we realize that it doesn't really matter what mass fraction we adopt as the constraint.

waves compatible with current limits on the number density of black holes given in Eq. (12) then becomes

$$\Omega_{\rm gw}^{\rm s} = \left(\frac{\epsilon_{\rm eff}}{3}\right) \frac{5\times 10^{-6}}{\Omega_0 h^2} \,. \tag{13}$$

That is, non-adiabatic perturbations lower the gravitational wave amplitude corresponding to the black hole limits.

B. String phase

We are assuming that the string phase has just the right properties to place the end of the dilaton phase into the observable window. Assuming efficient reheating ($T_{\rm reh} \sim 10^{18}$ GeV), this requires the string phase to be inflationary, with an expansion factor of about $T_{\rm reh}/10^9\,{\rm GeV} \sim 10^9$, since during radiation domination $aH \propto T$. The string phase must be inflationary for the gravitational waves generated in the dilaton phase to be detectable by LIGO, because otherwise the k^3 growth (relative to a scale-invariant spectrum) will lead to excessive black hole production on somewhat shorter scales, and also enough short-scale gravitational waves to disrupt nucleosynthesis [13]. However, the requirement that we see the end of the dilaton phase is not too unreasonable, since the LIGO sensitivity is not far from requiring that $A_{\rm T}$ be of order unity, a natural condition for string effects to become important.

We would then require an abrupt turn over in the spectrum to avoid large perturbations on shorter scales. Such behaviour has in fact been found in a toy model [21]. We therefore are in effect assuming a 'minimal' scenario, where it is assumed that significant perturbations are only generated in the dilaton phase.[‡]

If the expansion factor during the string phase exceeds 10^9, the frequency band accessible to the LIGO configuration would correspond to modes that went beyond the Hubble radius during this phase rather than the dilaton phase. The dynamics of this string phase where non-perturbative corrections are expected to become important is extremely uncertain. Gasperini [22] has shown that if the space–time curvature and kinetic energy of the dilaton field remain constant, the first–order corrections in the inverse string tension do not significantly affect the time evolution of the tensor perturbations, although the tilt of the spectrum may deviate from three due to the unknown behaviour of the scale factor.

Maggiore and Sturani [23] have attempted to describe the evolution of scalar perturbations through this era. In practice our calculation of the formation of black holes from density perturbations is not very sensitive to the precise tilt of the spectrum and merely assumes that the perturbations are growing towards a maximum at the scale $k_{\rm s}$. Similarly the relation between $A_{\rm T}^2$ and the logarithmic density $\Omega_{\rm gw}$ on a given scale used in Eq. (3) does not depend on the spectrum. However we have assumed that the ratio ϵ is exactly 3 (or less than 3 if we allow non-adiabatic perturbations). In all other known inflationary scenarios the effective value of ϵ is less than 3 and it is tempting to conjecture that 3 is the maximum possible value in any inflationary scenario. If so, the maximum density of gravitational waves compatible with black hole limits would remain that given by Eq. (12).

To produce $\epsilon > 3$ requires that we suppress scalar perturbations while still generating tensor perturbations. This seems to be difficult in standard theories of inflation, where one will always get perturbations in the field which controls the duration of inflation, but in the absence of any specific calculation for the perturbations in the string phase we cannot directly constrain the gravitational wave spectrum in terms of the scalar perturbations.

Finally, it is worth remarking that larger black holes with masses in the range $10^{-3}M_\odot \leq M \leq 1M_\odot$ could form by the mechanism outlined above, if the string phase is of the correct duration to place the end of the dilaton phase at the appropriate wavelength. This would have important implications for interpreting the microlensing events observed in our galaxy.

C. Reheating

The relation between modes leaving the horizon at the end of the dilaton phase and their comoving scale in the radiation-dominated era depends not only on the duration of the string phase, but also the reheat temperature at the start of the hot big bang. If reheating after the pre–big bang era is due to the decay of weakly coupled massive particles produced at the end of the string era, then the initial Hubble rate at the start of the hot big bang could be well below the string scale $M_{\rm st}$. The equation of state of an extended phase dominated by a massive scalar field undergoing coherent oscillations is effectively that of a pressureless fluid and this implies that $aH \propto t^{-1/3}$ [24]. This may reduce the required 10^9 expansion of the string phase. For example, if the universe is dominated by such a field between energy scales 10^{18}GeV and 10^9GeV, the inflationary expansion during the string phase should be 10^6 to place the end of the dilaton phase in the required range.

Black hole formation is not suppressed on frequencies above $f_{\rm s}$ if the spectrum of scalar perturbations generated during the string phase is flat or continues to grow on small scales. An extended mass spectrum of primordial black holes may form if the spectrum is precisely flat [15]. However, even a very small increase to-

[‡]The expression for the perturbations also assumes zero dilaton mass. Since this is disallowed in the present universe, it is usually argued that the dilaton acquires a mass at a relatively low energy scale such as the supersymmetry scale.

wards smaller scales implies that the mass spectrum will be dominated by the smallest black holes. This case may have interesting consequences for string cosmology. Black holes with masses as small as $M = \mathcal{O}(m_{\rm Pl}^2/M_{\rm st})$ could then form, where $m_{\rm Pl} \approx 10^{-5}$g is the Planck mass, and in most supersymmetric grand unified theories, $10^{-2} < M_{\rm st}/m_{\rm Pl} < 10^{-1}$ [25]. The only observational constraint below 10^4 g arises if black holes leave behind stable Planck mass relics in the final stages of their evaporation, but this now seems unlikely in view of the recent developments in the understanding of black hole evaporation in string theory (for a recent review, see, e.g. Ref. [26]). The copious production and rapid evaporation of black holes on these extremely small scales then provides a natural mechanism for black hole reheating of the universe after the string phase has ended [16,27,28].

D. Thermal Inflation

It is known that late entropy release, for instance from the decay of long-lived massive particles or evaporation of mini black holes, could suppress the present density of both black holes and gravitational waves [29]. However the constraint on the scalar perturbation amplitude is relatively insensitive to the number density of black holes, so late entropy release could only tighten the upper limit on the maximum amplitude of gravitational waves.

An extreme version of late entropy release is a second, relatively short, period of inflation known as 'thermal inflation' [30]. Thermal inflation has been proposed within the context of supersymmetric theories as a solution to a generic problem of inflationary models, such as the pre–big bang scenario, that have high reheat temperatures. This problem arises because moduli fields can come to dominate the universe before the onset of nucleosynthesis. Thermal inflation resolves this problem by diluting the moduli fields' energy density by a sufficient factor. If $a_{\rm i}$ and $a_{\rm f}$ denote the scale factors at the onset and end of thermal inflation respectively, then, like other massive relics, the density of black holes is diluted by a factor $(a_{\rm f}/a_{\rm i})^3$ relative to the radiation produced at the end of thermal inflation.

Thermal inflation is driven by a scalar field with vacuum expectation value $M \gg 10^3$GeV and mass corresponding to the supersymmetry scale, $m \approx 10^3$ GeV. Typically, it begins when the temperature falls below $T \approx (mM)^{1/2}$ and ends when $T \approx m$, so the expansion factor is $a_{\rm f}/a_{\rm i} \approx (M/m)^{1/2}$. Successful nucleosynthesis requires $M \leq 10^{14}$ GeV [31] and this implies that thermal inflation must begin below about 10^8 GeV.

Equation (4) requires modification if thermal inflation occurs, and one finds that the modes relevant to LIGO are within the Hubble radius throughout thermal inflation. This dramatically alters the density of gravitational waves on these wavelengths. Once gravitational waves have re-entered the Hubble length their energy density evolves like ordinary radiation. Although it is not diluted relative to other radiation in the standard radiation dominated era, it is diluted relative to the total energy density during thermal inflation. Thus, for scales which remain within the horizon throughout a period of thermal inflation, the energy density is diluted by a factor $(a_{\rm i}/a_{\rm f})^4 \approx (m/M)^2 \sim 10^{-22}$. Because we require that the initial amplitude $A_{\rm T}^2$ is less than unity, this dilutes the intensity of gravitational waves to way below the LIGO sensitivity.

This is quite a powerful and model-independent conclusion. Thermal inflation makes it impossible for LIGO to see a stochastic gravitational wave background generated in the very early universe.

V. SUMMARY

In the simplest pre–big bang scenario, a detection of gravitational waves at a high level implies that there should also be significant black hole production. There are many ways in which the simplest scenario may need amendment, such as nonadiabatic perturbations or late entropy production, and these changes reduce the maximum allowed gravitational wave amplitude. Therefore, if one were to detect gravitational waves at a high level, above that given by Eq. (12), without detecting black holes, it would suggest they were not produced in a dilaton driven pre–big bang phase.

Alternatively, one might detect both, but with the gravitational waves well below the result of Eq. (12). That would give an estimate of the importance of the various additional effects which would need to be incorporated into the scenario. However, if gravitational waves are not detected, this could be for any of several reasons and would not say anything much about the pre–big bang scenario.

To conclude, in the pre–big bang cosmology there are prospects for detection of both gravitational waves and black holes. Detection of the two in concert would provide strong supporting evidence for the pre–big bang scenario.

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